Relation of Interfacial Shear Stress to the Wave Height for Concurrent Air-Water Flow

LEMBIT U. LILLELEHT and THOMAS J. HANRATTY

University of Illinois, Urbana, Illinois

Waves at an interface between a concurrent air-water flow cause an increase in the interfacial stress. This increase in stress is correlated with the root-mean-square displacement of the liquid from its average height. The data are compared with Nikuradse's measurements with sand roughness.

Air flowing over a liquid surface generates waves, and these waves are responsible for a larger surface shear stress than would exist if the interface were smooth. A review of measurements of the interfacial stress in natural bodies of water and in water channels is presented by Ursell (*14*) and by Hicks and Whittenbury (*8*). Interest in two-phase flow problems encountered in engineering practice (*4, 9*) has prompted recent measurements of the interfacial stress for the concurrent flow of air and a liquid film (*2, 5, 7, 12*). Measurements of velocity profiles have shown that the velocity varies approximately as the logarithm of the distance from the surface as has been found for flows over roughened solid surfaces (*5*). Therefore it has been convenient to characterize the surface structure by comparing measurements of velocity profiles and surface stresses for water surfaces with those for roughened solid surfaces. Nikuradse measured the pressure drop in circular pipes covered on the inside as closely as possible with sand of a definite grain size glued to the wall. He found that three regimes could be defined depending on the size of the sand roughness k_s. For very small k_s the roughness had no effect on the surface stress $\left(\frac{k_s u^*}{v_g} < 5\right)$. Such surfaces are called *hydraulically smooth.* When the k_s is large enough that the viscosity of the fluid does affect the surface stress, the surface is called *completely rough* $\left(\frac{k_s u^*}{v_g} > 70\right)$. In the transition regime both the surface roughness and the fluid viscosity are important $\left(5 \leq \frac{k_s u^*}{v_g} \leq 70\right)$. Other roughened surfaces show these same regimes; however the functionality between the surface stress and the roughness size k and the limits of the three regimes can be quite different from those obtained by Nikuradse in his sand roughness experiments. Nevertheless the roughness of other surfaces has been characterized by comparing surface shear stress measurements in the completely rough regime with those of Nikuradse and by defining an equivalent sand roughness. This equivalent sand roughness can be quite different from the actual size of the roughness elements (*13*). Therefore the definition of equivalent sand roughness for gas liquid interfaces is completely arbitrary and might bear little relation to the actual wave structure. This is especially true if the comparison with Nikuradse's data is made in the intermediate range. The problem is further complicated by the fact that unlike solid surfaces the interfacial structure can change with the flow rate of the gas. This paper presents the results of experiments performed to find the relation between the equivalent sand roughness k_s and the actual interfacial structure. This interfacial structure will be characterized by the root-mean-square displacement of the interface from the average liquid height. The double of this root-mean-square displacement would be a measure of the height of the waves from crest to trough. Some preliminary measurements are also presented on the effect of flow variables on the root-mean-square displacement. The measurements were made for the concurrent flow of air and a thin water film in an enclosed channel. The system differs from that encountered in meteorological studies in that the structure of the interface depends on the height of the film and that the range of wave heights and air velocities are much smaller. Nevertheless since the small scale wave structure on large bodies of water is believed to be playing an important role in affecting the shear stress, the results presented in this paper might be helpful in understanding the surface stress data obtained in meteorological studies.

Lembit U. Lilleleht is with University of Alberta, Edmonton, Alberta.

THEORY

Nikuradse's velocity profile measurements for roughened surfaces may be described in terms of the dimensionless groups $\frac{u}{u^*}$, $\frac{y}{k_s}$, and $\frac{k_s u^*}{v_g}$ by

$$\frac{u}{u^*} = 5.75 \log \frac{y}{k_s} + B \qquad (1)$$

where the quantity B is a function of $k_s u^*/v_g$. For the hydraulically smooth region Nikuradse's data gave

$$B = 5.5 + 5.75 \log \frac{u^* k_s}{v_g} \qquad (2)$$

$$\frac{u}{u^*} = 5.75 \log \frac{y u^*}{v_g} + 5.5 \qquad (3)$$

If Equation (1) is subtracted from Equation (3) and the difference is denoted by $\Delta(u/u^*)$, there results

$$\Delta\left(\frac{u}{u^*}\right) = 5.5 + 5.75 \log \frac{k_s u^*}{v_g} - B \qquad (4)$$

A plot of B as a function of $k_s u^*/v_g$ is given by Schlichting (*13*) in Figure 20.20 of his book. Nikuradse's measurements showed that the quantity $\Delta\left(\frac{u}{u^*}\right)$ is a function of $k_s u^*/v_g$ for the sand roughened surfaces with which he carried out his measurements, and the functionality is given by Equation (4).

The height term used to characterize the interfacial roughness in this paper is defined as

$$2\Delta h' = 2\sqrt{\overline{(h - \overline{h})^2}}$$

As a first approximation the measured velocity profiles over interfaces will be correlated with an equation analogous to Equation (4). Measured values of $\Delta\left(\frac{u}{u^*}\right)$ are correlated as a function of $2\Delta h' u^*/v$.

If it is assumed that the only influence that gas flow has upon the surface

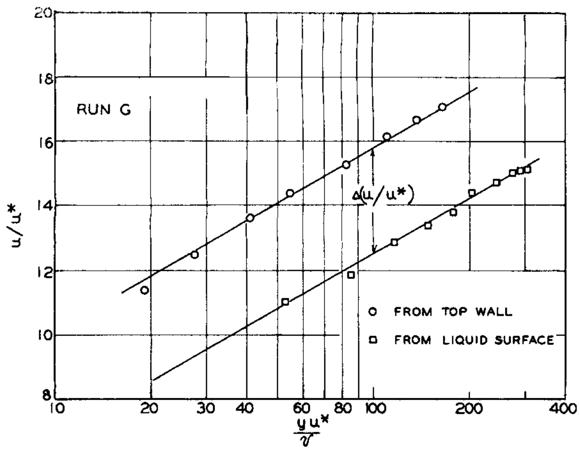

Fig. 1. Velocity profile measurements.

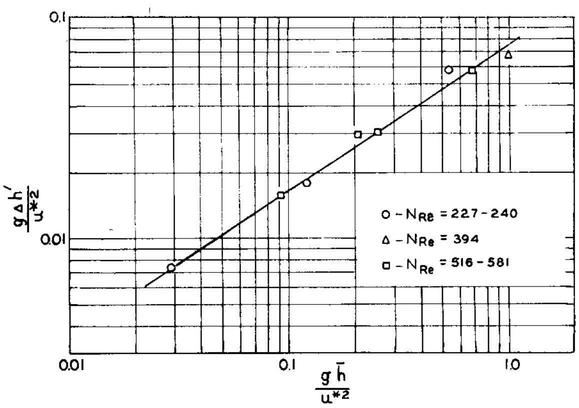

Fig. 3. Interfacial displacement data.

structure is through the friction velocity u^*, then the following functionality might be assumed:

$$\Delta h' = f\,(\overline{h}, g, u^*, \nu, \sigma) \qquad (5)$$

The liquid velocity is not used, since for the systems studied it is fixed for given values of u^*, $\overline{h}$, and ν. If ν and σ were not important, then

$$\frac{g\Delta h'}{u^{*2}} = f\left(\frac{g\,\overline{h}}{u^{*2}}\right) \qquad (6)$$

For very large depths such as are encountered in natural bodies of water $\overline{h}$ would not be an important variable, and

$$\frac{g\Delta h'}{u^{*2}} = \text{constant} \qquad (7)$$

Ellison (3), in discussing Hay's (6) measurements over a completely rough sea, showed that equivalent sand roughness measurements could be correlated by an equation analogous to Equation (7):

$$\frac{g\,k_s}{u^{*2}} = 2.28$$

EXPERIMENTAL

Measurements were carried out in an enclosed horizontal 20 ft. long Plexiglas channel 1 in. high by 1 ft. wide. A water film flowed along the bottom of the channel, and air was blown over the water surface. Pressure taps along the top of the channel allowed the measurements of the pressure drop. Velocity profile data were obtained with an impact tube at a location far enough along the channel such that the velocity profile was fully developed and that there were no noticeable changes in interfacial structure in the direction of flow. The film height was measured by passing a small chopped beam of light through the channel and the liquid film on to a photomultiplier tube. By dissolving methylene blue dye in the water the liquid could be made to absorb an appreciable amount of light. Variations in the thickness of the liquid film then resulted in variations in the intensity of the light impinging on the photomultiplier tube. The output from the photomultiplier tube was amplified and demodulated. The resulting signal was recorded on a Visicorder, and the root-mean-square value of the a.c. component of the signal was measured with a random signal voltmeter. Details of the technique and of the equipment in which measurements were made and a discussion of experimental errors are given in a thesis by one of the authors (10) and in another paper (11).

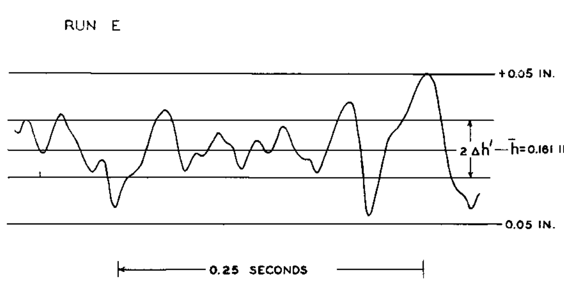

Fig. 2. Water film height as a function of time.

TREATMENT OF DATA

Measured velocity profiles over a roughened liquid surface were unsymmetrical as reported in a previous paper (5). The maximum in the velocity profile was displaced away from the roughened surface. By locating the maximum the shear stress at the dry channel surface and at the interface could be calculated with the pressure-drop measurement and the measurement of the average film height:

$$\tau_s = -\,a\,\frac{dp}{dx} \qquad (8)$$

$$\tau_o = -\,(b - \overline{h} - a)\,\frac{dp}{dx} \qquad (9)$$

Velocity data for one of the runs are presented in Figure 1 plotted as u/u^* vs. yu^*/ν_g, where y may be the distance from the top of the channel or the distance from the interface and the values of u^* are based on either τ_s or τ_o. The top portions of the velocity profiles were representative of the smooth channel velocity measurements, while the bottom portions were displaced downward from and were approximately parallel with the top portions. The quantity $\Delta(u/u^*)$ was measured as indicated in Figure 1. Although

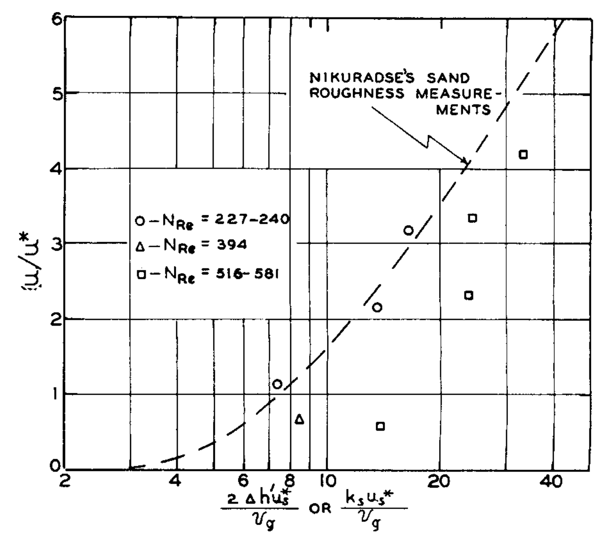

Fig. 4. Relation of $\Delta u/u^*$ to surface roughness.

TABLE 1. SUMMARY OF EXPERIMENTAL CONDITIONS

$b = 0.0850$ ft.
$\nu_g = 1.76 - 1.82 \times 10^{-4}$ sq. ft./sec.
$\nu = 0.92 - 1.04 \times 10^{-5}$ sq. ft./sec.

Run designation	N_{Re}	N_{Re_g}	$\overline{h}$, $10^3 \times$ ft.	$\Delta h'$, $10^3 \times$ ft.	u^*_s, ft./sec.	u^*_o, ft./sec.	$\Delta u/u^*$
A*	239	4,700	8.42	0.910	0.714	0.684	1.17
B	227	8,420	6.19	0.925	1.285	1.179	2.18
C	394	4,350	15.8	1.05	0.709	0.691	0.67
D	550	4,930	16.7	1.38	0.889	0.866	0.59
E	530	7,420	13.4	1.61	1.313	1.197	2.34
F	516	10,560	9.42	1.62	1.820	1.561	4.21
G	581	7,300	11.25	1.61	1.329	1.160	3.38
H	240	12,280	3.17	0.80	1.866	1.661	3.20

* Two-dimensional waves.

there appeared to be a small variation in the slope of the velocity profile data from different runs when plotted in the form presented in Figure 1, a line of slope 5.75 appeared satisfactorily to represent all of the data at $yu^*/\nu_g >$ 30 within the limits of experimental error. The smooth channel data showed a variation with Reynolds number of the constant B appearing in Equation (1) similar to that reported by Corcoran and Sage (1).

RESULTS

One of the runs (A) was made with an interface having two-dimensional waves (5). All the other runs were with three-dimensional waves (5). A plot of film height vs. time for the three-dimensional waves is shown in Figure 2. The height used in this paper to characterize the interfacial roughness $2\Delta h'$ is indicated in the figure. These are measurements of the film height at a fixed position in the channel. Since the wave lengths are of the magnitude of 0.20 in., the surface structure at any given time would exhibit approximately the same height distribution; however the surface slopes would be much less than indicated in Figure 2. The time axis should be stretched out about sevenfold to give a representation of the interfacial structure. The ratio of root-mean-square displacement to the average height $\Delta h'/\overline{h}$ varied for the runs reported from 0.067 to 0.253. A summary of the experimental conditions is presented in Table 1. The measurements of $\Delta h'$ are plotted in Figure 3. The plot of $g\Delta h'/u^{*2}$ vs. $g\overline{h}/u^{*2}$ brings the data for different liquid Reynolds numbers together on a single line. However data were obtained over too small a range of variables to determine the effect of ν and σ, and the fact that these two fluid properties do not appear in the plot in Figure 3 does not necessarily imply that they do not play an important role in determining $\Delta h'$. However the data do show a marked dependence of $\Delta h'$ on the average film height $\overline{h}$. It is interesting to compare the values of $g\Delta h'/u^{*2}$ in Figure 3 with the value of $gk_s/u^{*2} = 2.28$ obtained from Hay's measurements over an open sea.

The values of $\Delta(u/u^*)$ determined from the velocity profiles are plotted against $2\,\Delta h'\,u^*/\nu$ in Figure 4 along with the line for Nikuradse's sand roughness data calculated from Equation (4). The wave roughness measurements behave similarly to the sand roughness measurements of Nikuradse and appear to be in the intermediate roughness range as defined by his data. The spread of the data would indicate that the effect of interfacial roughness on the shear stress is not uniquely defined by the parameter $2\,\Delta h'\,u^*/\nu_g$. However with the exception of two of the measurements shown on Figure 4 the interfacial roughness parameter $2\Delta h'$ appears to be of the same magnitude as k_s.

ACKNOWLEDGMENT

The authors are grateful to the American Oil Company and to E. I. duPont De Nemours & Company for financial support.

NOTATION

a = distance from the maximum in the velocity profile to the interface
b = height of the channel
B = function plotted in Figure 20.20 of Schlichting's book
g = acceleration of gravity
h = instantaneous height of the liquid film
$\overline{h}$ = time average height of the liquid
$\Delta h'$ = $[\overline{(h-\overline{h})^2}]^{1/2}$
k = size of a roughness element
k_s = size of a Nikuradse sand roughness
N_{Re} = liquid Reynolds number equal to W/μ
N_{Re_g} = gas Reynolds number equal to W_g/μ_g
p = pressure
u = gas velocity
u^* = friction velocity equal to $(\tau_s/\rho)^{1/2}$ or $(\tau_o/\rho)^{1/2}$
Δu = displacement of the velocity profile plot due to surface roughness
W = weight rate of flow of the liquid
W_g = weight rate of flow of the gas
x = distance in the direction of flow
y = distance from the top wall or from the liquid-gas interface

Greek Letters

μ = liquid viscosity
μ_g = gas viscosity
ν = kinematic viscosity of the liquid
ν_g = kinematic viscosity of the gas
σ = interfacial tension
τ_s = stress at the interface
τ_o = stress at the top wall of the channel

LITERATURE CITED

1. Corcoran, W. H., and B. H. Sage, *A.I.Ch.E. Journal*, **2**, 251 (1956).
2. Ellis, S. R. M., and B. Gay, *Trans. Inst. Chem. Engrs. (London)*, **37**, 206 (1959).
3. Ellison, T. H., "Surveys in Mechanics," p. 400, Cambridge Univ. Press, England (1956).
4. Gresham, W. A., P. A. Foster, and R. J. Kyle, *Eng. Exp. Station Georgia Tech. WADC Tech. Rept. 55-422* (1955).
5. Hanratty, T. J., and J. M. Engen, *A.I.Ch.E. Journal*, **3**, 299 (1957).
6. Hay, J. S., *Quart. J. R. Met. Soc.*, **81**, 307 (1955).
7. Hershman, Arnold, Ph.D. thesis, Univ. Illinois, Urbana, Illinois (1960).
8. Hicks, Bruce L., and Clive G. Whittenbury, *Rept. R-83*, p. 90, Control Systems Laboratory, Univ. Illinois, Urbana, Illinois (1956).
9. Kinney, G. R., A. E. Abramson, and J. L. Sloop, *Natl. Advisory Comm. Aeronaut. Tech Rept. 1087* (1952).
10. Lilleleht, L. U., Ph.D. thesis, Univ. Illinois, to be written.
11. ———, and T. J. Hanratty, *J. Fluid Mechanics*, to be published.
12. van Rossum, J. J., *Chem. Eng. Sci.*, **11**, 35 (1959).
13. Schlichting, Hermann, "Boundary Layer Theory," pp. 416-426, McGraw-Hill, New York (1955).
14. Ursell, F., "Surveys in Mechanics," p. 240, Cambridge Univ. Press, England (1956).

Manuscript received October 3, 1960; revision received April 10, 1961; paper accepted April 11, 1961.